Department of Mathematics, East China Normal University

Mid-term Examination Session 2010-2011

Advanced Ordinary Differential Equations (Required)

16-23 November 2010 (Exam)

Examinee ______________

Registration Number ____________

Specialty ______________

Total Marks	
Instructor's Signature	

Note: Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer all four questions. ALL questions carry equal weight.

Question 1. Consider $\dot{x} = f(t,x) = -(1+x^2)x + e^t$ with $x(0) = x_0$.

1) Verify the conditions on f for which there exists $\omega_+ > 0$ such that the above IVP has a unique solution $x(t)$ on $[0, \omega_+)$;

2) Let $v(t) = x^2(t)$. Show that $v'(t) \le -2v(t) + 2\sqrt{v(t)}\, e^t$ on $[0, \omega_+)$;

3) Solve the Bernoulli equation $u' = -2u + 2\sqrt{u}\, e^t$ with $u(0) = (x_0)^2$;

4) Find the bound of $x(t)$ with $x(0) = x_0$ on $[0, \omega_+)$;

5) Show that $\omega_+ = +\infty$.

Question 2. Consider $x' = f(t,x)$ with $x(t_0) = x_0$, where $f(t,x)$ is continuous, locally Lipshitz in x and

$$\| f(t,x) \| \le b + a \| x \|, \quad \forall (t,x) \in [t_0, \infty) \times R^n, \quad a \ne 0.$$

1) Using the comparison lemma to show that the solution $x(t, t_0, x_0)$ of the IVP satisfies

$$\| x(t,t_0,x_0) \| \le \| x_0 \| e^{a(t-t_0)} + \frac{b}{a} \cdot [e^{-a(t-t_0)} - 1]$$

for all $t \ge t_0$.

2) Is it possible for $x(t,t_0,x_0)$ to have a blow up at finite time? Give reasons please.

Question 3. Suppose that $x_1(t)$, $x_2(t), \cdots$ and $x_{n+1}(t)$ defined on I are $n+1$ linearly independent solutions of

$$\frac{dx}{dt} = A(t)x + h(t),$$

where $A(t)$ and $h(t) \in C(I)$, $h(t)$ is not identically zero on I.

1) Write down its general solution;
2) Show that it is really a general solution.

Question 4. Using Decomposition Theorem to solve $x' = Ax$ with $x(0) = x_0$, where

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 2 & 0 & 1 & 0 \end{pmatrix}.$$